B. A. Taking the Ø-Bin into Account

w/ A. Manohar

Outline

Motivation

Double Counting, $\sum_{p\neq 0}\int d^4x$, Thr. Expn

Pull-up hep-ph/0102257 [predates $\mathcal{L}_{SCET}$]

Hoang, I.S., Stewart

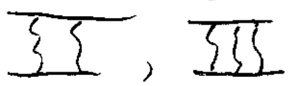
Ø-bin

Examples: I Ø-bin ⇒ pull-up

II resolves pinch sing. issue } NRQCD

[unphysical]

pinches in soft boxes



$$\int \frac{dk^0}{(k^0+i\epsilon)(-k^0+i\epsilon)}$$ ← problem in Thr. Expn

[soft mode can't be defined]

III $SCET_I$ loops

IV tree level

V $SCET_{II}$ implications for endpt. sing. [no unphysical ...]

pinch surfaces

Coleman-Norton

$$\int_0^1 dx \, \frac{\phi_\pi(x)}{x^2}$$

[No change to ~~cases~~ that work.]

Facts

- EFT is indep. of IR regulator choice [d.o.f.]
- Poor IR ($\neq$ UV) regulators can spoil p.c. [not an issue]
- $$\int \frac{d^d k}{k^4} = \frac{i}{16\pi^2}\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)$$
  requires δZ

Motivation

- Formulate NRQCD, SCET indep. of dim. reg. (UV) and IR regulator
- Solve dble counting, pull-up, singularity issues

Lamb Shifts, $t\bar{t}$ production
Static Scattering
$B \to \pi \ell \bar{\nu}$
$B \to \pi\pi$ (annih.)
$\gamma^* \pi \to \pi$ (subl.)
$\gamma^* p \to p$ (F_2)
$\gamma^* \to q\bar{q}\, g$
$B^0 \to D^0 K^0$
⋮

– Insert picture from (4) –

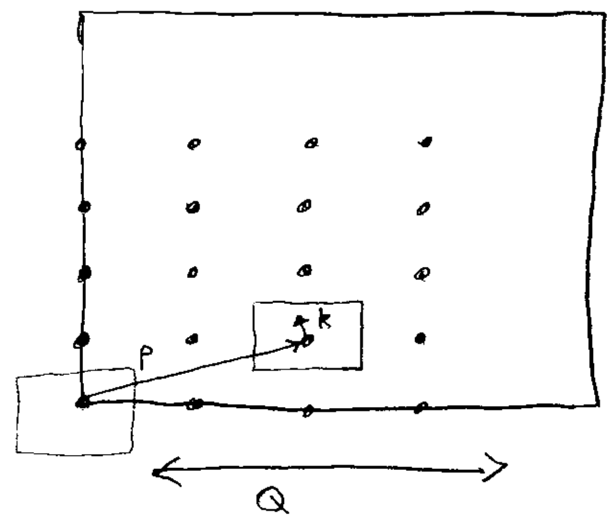
– Why two low energy d.o.f. ? p.c.

Must capture all IR scales $m_1, m_2, \ldots$

[Sometimes 1 d.o.f makes p.c. so opaque that its impossible to setup the theory]

$$\psi(x) = \sum_p e^{-ip\cdot x}\, \varphi_p(x)$$

collinear $P = p + p_r$
label residual

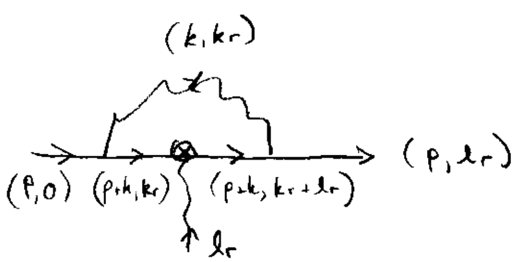


- manifest p.c.
- homo. derivatives

Note: $\sum_{p\neq 0}$ otherwise P is soft !

this is the zero-bin

Loop



[LMR]

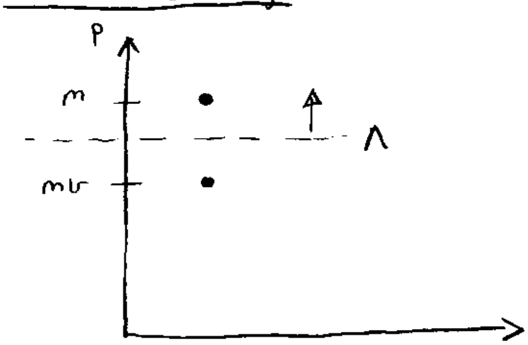
$$\sum_k \int dk_r = \int dk$$

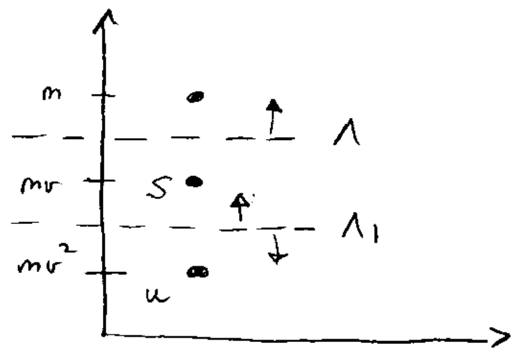
N_0

leave l_r

[later, but formulation at level of int. is incorrect.

Dble. Counting





cutoffs

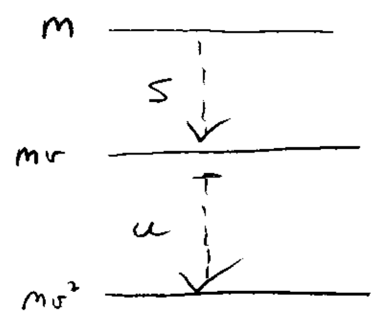
Scaless Reg.

[dble counting in UV taken care of by W. Coeff.]

$\Lambda_1 \to 0$ for s double counts IR with UV of u

$$I_s = \frac{A}{\epsilon_{UV}} + \frac{B}{\epsilon_{IR}^{mv}} + f\left(\frac{s}{\mu}\right)$$

$$I_u = \frac{-B}{\epsilon_{UV}^{mv}} + \frac{C}{\epsilon_{IR}^{mv^2}} + g\left(\frac{u}{\mu}\right)$$



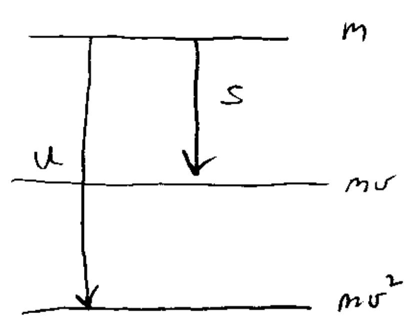
one mode at a time

[QFT, c.t.

wrong with both modes

- doesn't reproduce IR
- doesn't run simult. from UV [modes not formulated simult.]

Instead



Adds $B\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right)$ to s

subtract from u

$$I_s = \frac{A+B}{\epsilon_{UV}} + \dots$$

by hand

$$I_u = \frac{-B}{\epsilon_{UV}} + \frac{C}{\epsilon_{IR}}$$

pulled up to hard scale

Strong Check: NLL NRQED Anom. Dim

$\alpha^8 \ln^3 \alpha$	Lamb	H, e^+e^-
$\alpha^7 \ln^2 \alpha$	Lamb, h.f.s	"
$\alpha^3 \ln^2 \alpha$	$\Delta\Gamma$	e^+e^-

$\emptyset$- Bin Subtraction:

unique

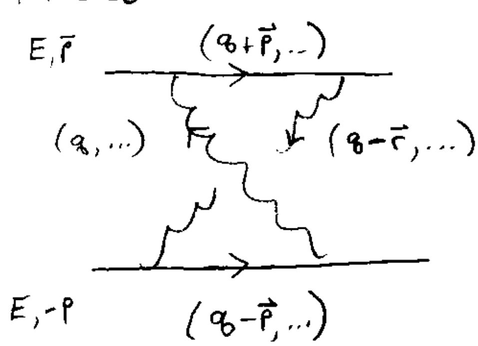
$$\sum_{\{p_i \neq 0\}} \int \prod_i dk_i \; F(p_i, k_i) \;\rightarrow\; \int \prod_i dp_i \Big[F(p_i) - \sum_{j \in \mathcal{U}} F_j^{sub}(p_i) \Big]$$

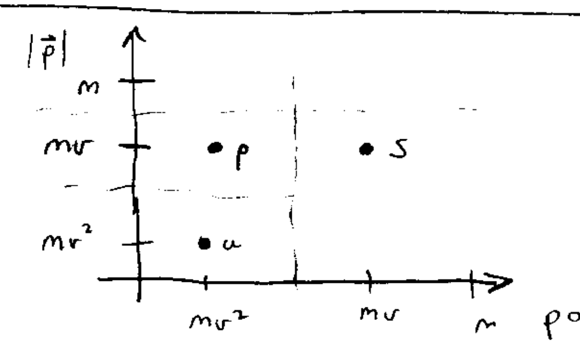
determined by labelled field contractions

but with mom. conservation it acts on entire loop integral

removes support of integrand to avoid dble. counting!

NRQCD





[soft - usoft gluons
[ptnl - soft quarks

$$I_s = \sum_{\substack{q^0 \neq 0 \\ q^\mu \neq 0 \\ q^\mu \neq \vec{r}}} \int d^d q \; \frac{1}{(q^0 + i\epsilon)(q^0 + i\epsilon)(q^2)(q - \vec{r})^2}$$

$$I_s = \tilde{I}_s - I_1 - I_2$$

naive

$$\tilde{I}_S = \int \frac{d^dq}{\text{Pen}}$$

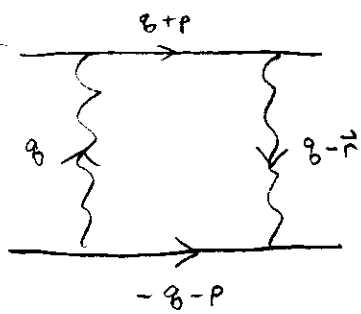
$$I_1 = \int \frac{d^dq}{(q^0)(q^0)\;(q^2)\;(-\vec{r}^{\,2})} \qquad \leftarrow \quad q^\mu = 0$$

$$I_2 = \int \frac{d^dq}{(q^0)(q^0)\;(-\vec{r}^{\,2})\;(q-\vec{r})^2} \qquad \leftarrow \quad q^\mu = \vec{r}$$

$$\tilde{I}_S = \frac{1}{\epsilon_{IR}} + \ln\left(\mu^2/\vec{r}^{\,2}\right)$$

$$-I_1 - I_2 = -\frac{1}{\epsilon_{IR}} + \frac{1}{\epsilon_{UV}}$$

✓ <u>Pull-up</u> (I)



C-box arose ↓

$$I_S = \sum_{\substack{q^0 \neq 0 \\ q^\mu \neq 0 \\ q^\mu \neq \vec{r}}} \int dq_r \frac{1}{(q^0+i\epsilon)(-q^0+i\epsilon)\;(q^2)\;(q-\vec{r})^2}$$

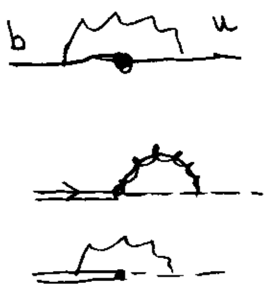
$$I_S = \underbrace{\tilde{I}_S - I_1 - I_2}_{\text{like before}} \;\underbrace{-I_3}_{p^0=0} + \underbrace{I_{13} + I_{23}}_{p^0=0 \text{ limit of } I_1,\, I_2}$$

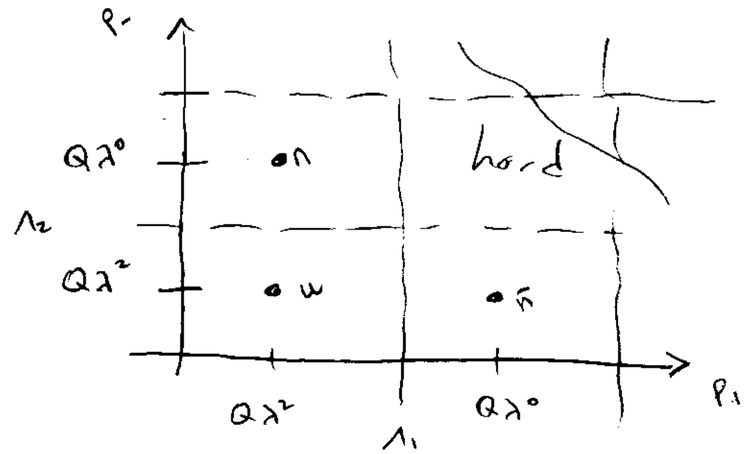
$$I_3 = \int \frac{d^dq}{(q^0+i\epsilon)(-q^0+i\epsilon)\;(-\vec{q}^{\,2})\;(-(\vec{q}-\vec{r})^2)}$$

$$\left.\begin{array}{l} \tilde{I}_S - I_3 \\ I_1 - I_{13} \\ I_2 - I_{23} \end{array}\right\} \text{ have no pinch!}$$

[Equivalent to "ignore the pinch" etc.]

SCET$_{\text{I}}$





n-collin vs. usoft

Check IR [diff. regulators]

$$I_{full} = \int \frac{d^d q \quad 4 p_b \cdot p}{(q^2)\,(q^2 + 2 p_b \cdot p + i\epsilon)\,(q^2 + 2 p \cdot q)}$$ on-shell

$$= \int_{-p^-}^{0} dq^- \, d^n q_\perp \, \frac{q^-}{(q_\perp^2 + q^{-2})(q_\perp^2)} + \text{finite}$$

$$= Li_2\left(\frac{-\Omega_\perp^2}{\Omega_-^2}\right) + \ln\left(\frac{\Omega_-}{p^-}\right) \ln\left(\frac{-l - p_-}{\Omega_\perp^2}\right) + \ldots$$

$$\Lambda_\perp^2 \geq q_\perp^2 \geq \Omega_\perp^2$$
$$\Lambda_-^2 \geq q_-^2 \geq \Omega_-^2$$

$$I_c = \sum_{\tilde{q} \neq 0} \int d^4 q_r \frac{2\,\bar{n}\cdot(p + \cancel{q})}{(\bar{n}\cdot q)\,(q^2 + 2p\cdot q)\,(q^2)}$$

$$\tilde{I}_c = \int d^d q \, (\ldots) = \int_{-p^-}^{0} dq^- \, d^n q_\perp \frac{2 p^-}{(q^-)\,(p^-)\,(q_\perp^2)}$$ $q^- = 0$ is IR div.

$$I_1 = \int \frac{d^d q \quad 2\,\bar{n}\cdot p}{(\bar{n}\cdot q)\,(\bar{n}\cdot p \, n\cdot q)\,(q^2)} = \int_{-\infty}^{0} (\,''\,)$$

$\boxed{\text{indep. of cutoffs}}$

$$\tilde{I}_c - I_1 = \int_{-p^-}^{-\infty} (\,''\,)$$ no $q^- = 0$ div!

$$\tilde{I}_c = -\ln\left(\frac{\Omega_\perp^2}{\Lambda_\perp^2}\right)\ln\left(\frac{\Omega_-}{p^-}\right) + \cdots$$

$$-I_1 = +\ln\left(\frac{\Omega_\perp^2}{\Lambda_\perp^2}\right)\ln\left(\frac{\Omega_-}{\Lambda_-}\right) + \cdots$$

$$I_{us} = \mathrm{Li}_2\left(\frac{-\Omega_\perp^2}{\Omega_-^2}\right) + \ln\left(\frac{\Omega_-}{\Lambda_-}\right)\ln\left(\frac{\Omega_-\Lambda_-}{\Omega_\perp^2}\right) + \cdots$$

$$\text{sum} = \underbrace{\mathrm{Li}_2(\,) + \ln\left(\frac{\Omega_-}{p^-}\right)\ln\left(\frac{\Omega_- p_-}{\Omega_\perp^2}\right)}_{\text{IR agrees}} + \underbrace{\ln^2\left(\frac{\Lambda_-}{p^-}\right) - \ln^2\left(\frac{\Lambda_-}{\Lambda_-}\right)}_{\text{UV}}$$

need I_1 else

$\ln\left(\frac{\Omega_\perp^2}{\Lambda_\perp^2}\right)\ln\Omega_-$ spoils it

[Also needed so no IR × UV cross term for c.t.]

<u>Offshellness</u> $p^+ \neq 0$

$$I_{full} = \ln^2\left(\frac{-p^+}{p^-}\right)$$

$$I_{us} = \left(\frac{1}{\epsilon_{uv}^2} + \frac{2}{\epsilon_{uv}}\ln\left(\frac{\mu}{-p^+}\right) + 2\ln^2\left(\frac{\mu}{-p^+}\right)\right)$$

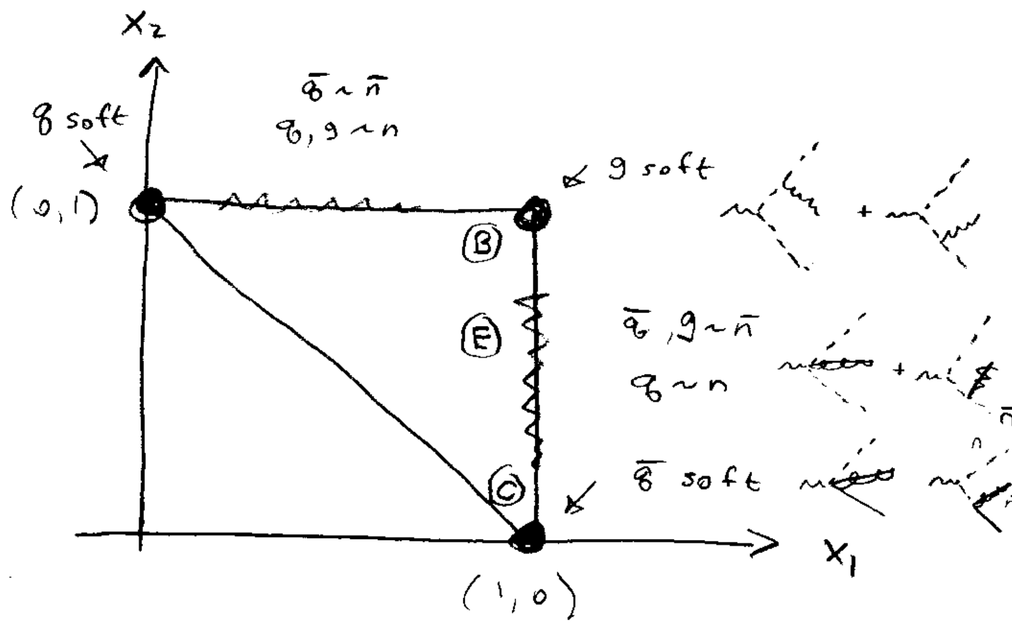
$$\tilde{I}_c = \left(\frac{-2}{\epsilon_{uv}\epsilon_{IR}} - \frac{2}{\epsilon_{IR}}\ln\left(\frac{\mu}{-p^2}\right) - \ln^2\left(\frac{\mu^2}{-p^2}\right) + \left(\frac{2}{\epsilon_{IR}} - \frac{2}{\epsilon_{uv}}\right)\ln\left(\frac{\mu}{p^-}\right)\right)$$

$$-I_1 = \left(\left(\frac{2}{\epsilon_{IR}} - \frac{2}{\epsilon_{uv}}\right)\left(\frac{1}{\epsilon_{uv}} + \ln\left(\frac{\mu^2}{-p^2}\right) - \ln\left(\frac{\mu}{p^-}\right)\right)\right)$$

$$I_{EFT} = \frac{-1}{\epsilon_{uv}^2} - \frac{2}{\epsilon_{uv}}\ln\left(\frac{\mu}{p^-}\right) - 2\ln^2\left(\frac{\mu}{p^-}\right) + \ln^2\left(\frac{-p^+}{p^-}\right)$$

$\gamma^* \to q\,\bar{q}\,g$
$\quad\;\;\; 1\;\; 2\;\; 3$

$x_i = \dfrac{2E_i}{Q}$



full cross section
$$\frac{d\sigma}{\sigma_0\, dx_1 dx_2} = \frac{2\alpha_s}{3\pi}\left[\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\right]$$

B $\dfrac{2}{(1-x_1)(1-x_2)}$ $\quad x_1, x_2 \sim 1$

E $\dfrac{x_2^2+1}{(1-x_2)(1-x_1)}$ $\quad x_2 \not\sim 0,\ x_2 \not\sim 1,\ x_1 \sim 1$

C $\dfrac{1}{(1-x_1)}$ $\quad x_2 \sim 0,\ x_1 \sim 1$

at tree level implemented by $\sum_{p \neq 0}$ in SCET

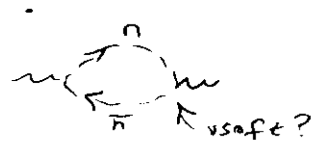
no dble counting

↑ p.c. as expected

pick an x_1, x_2, decide where you are

What if I want to add them?

can't use loops ie thr expn.

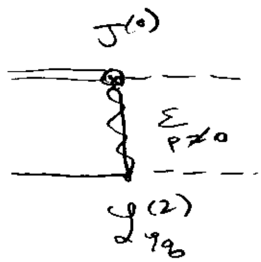


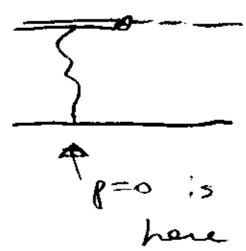
$$\int \frac{d^dk}{n(k+p)\,\bar{n}(k-p)} = 0$$

can square m.elt! but must implement subtractions in phase space. then sum gives full theory results

So Subtractions are needed at tree level when we integrate over ext. momenta & are unique

e.g. $B \to \pi \ell \nu$ in $SCET_I$





no endpt sing.

Agrees with QCD

Match $\sum_{p \neq 0}$ onto $SCET_{II}$

Homework:
- repeat for $SCET_{II}$
- Fourier transform to get position space formulation of $\sum_{p \neq 0}$